The Cathedral School of St Peter and St John RC Primary

Growing together within God's loving family, we support, challenge and inspire each other to become the best that we can be.'

Mathematics Calculation Progression Policy

September 2019

The National Curriculum:

The National Curriculum (2014) sets out the end of year expectations in mathematics.

This document provides a list of expectations, by year group showing the suggested progression of skills using the Concrete-Pictorial-Abstract approach (Maths No Problem).

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. … pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.
National Curriculum 2014

The national curriculum for mathematics aims to ensure that all pupils:

• become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

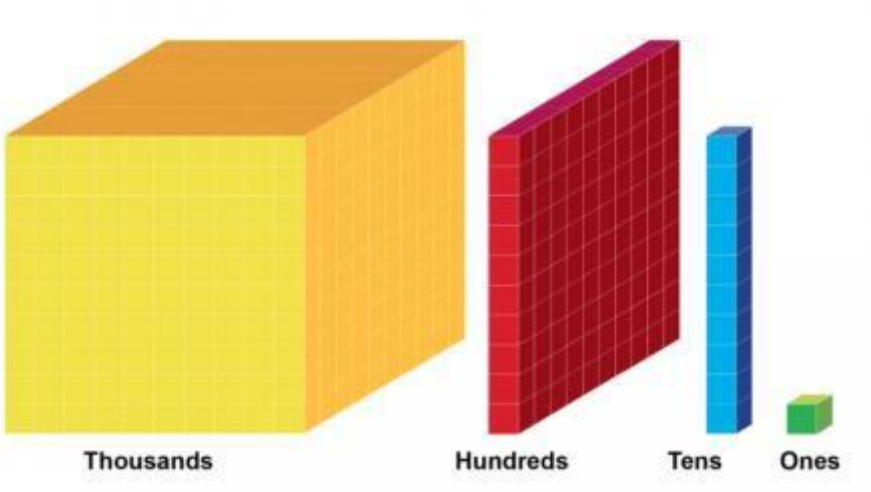
• reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

• can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

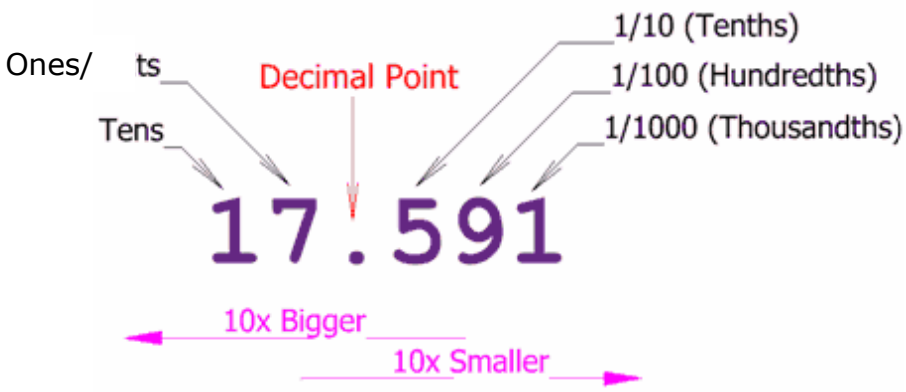
PLACE VALUE

Here at the Cathedral School a good understanding of place value (the value of each digit in a number) is vital in maths. Place Value is defined as: the value of where the digit is in the number.

Example: In 352, the 5 is in the "tens" position, so it shows a value of 50



Example: In 17.591, the 9 is in the "hundredths" position, so it shows a value of 0.09

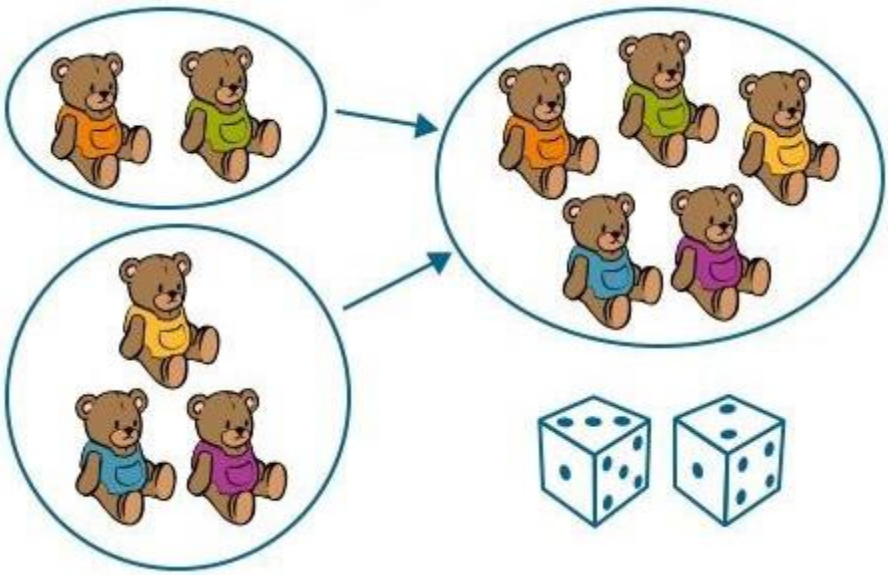


ADDITION

Addition is not simply 2+2 = 4. Children build on their knowledge of place value by encountering a range of different addition problems. Addition is finding the total, or sum, by combining two or more numbers. However, it can be categorised in a range of different ways.

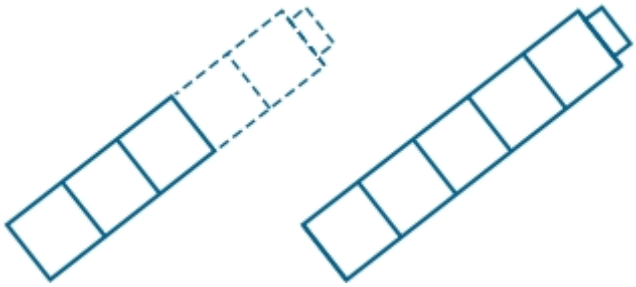
Aggregation:

Combining two or more quantities.



How much altogether? How many? What is the total?

2. Augmentation:

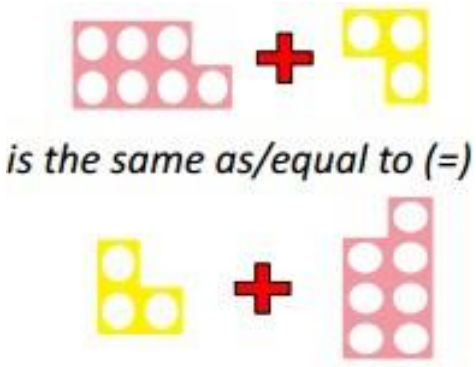


A quantity is increased by a specified amount.

Start at and count on, increase by, go up by.

Commutative law:

Addition can be done in any order. Start with bigger number when counting on



Children to be made aware that subtraction does not share this property.

ADDITION AND SUBRACTION EXPECTATIONS BY YEAR GROUPS

Year 1:

Related statutory requirements

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, leas
- read and write numbers from 1 to 20 in numerals and words.
- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 9 = □ + 3.

Year 2:

Related statutory requirements

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use <, > and = signs

- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems.
- solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens ; two two-digit numbers
- adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Year 3:

Related statutory requirements

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas.
- add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 4:

Related statutory requirements

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000

- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- Children should extend the carrying method to numbers with at least four digits and decimals

Year 5:

Related statutory requirements

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

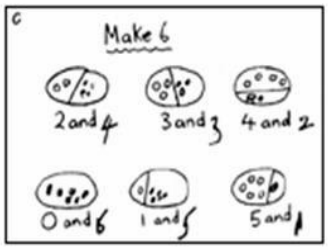
Year 6

Children should have a range of mental and written methods, including formal columnar methods, which they can apply efficiently. They should have opportunities to apply these through varied and frequent practice with increasingly complex problems.

PROGRESSION THROUGH CALCULATION STRATEGIES

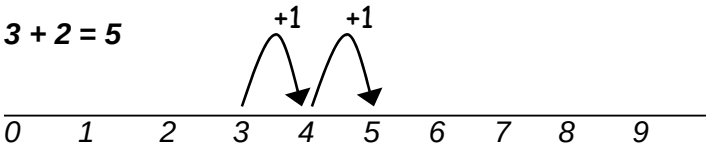
Year 1:

Children develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

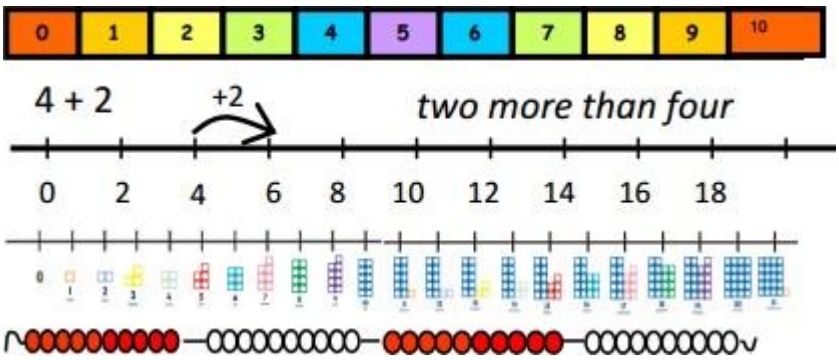


They use number lines and practical resources to support calculation and teachers demonstrate the use of the number line involving children in practical demonstrations.

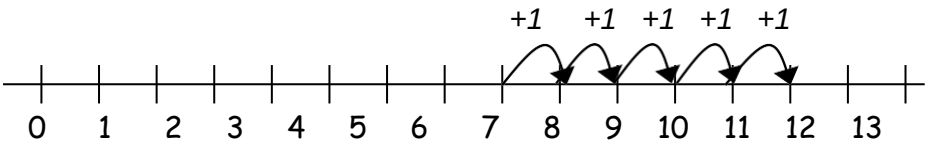
3 + 2 = 5



Children then begin to independently use number lines to support their own calculations using a number line to count on in ones.

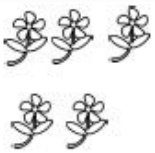
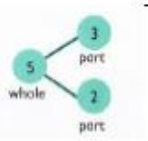
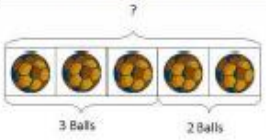
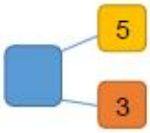


7 + 5 = 12

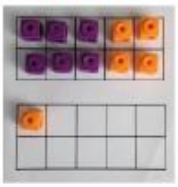
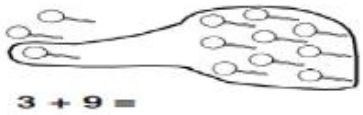
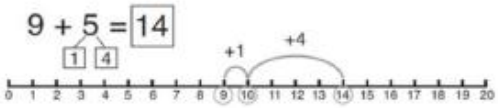


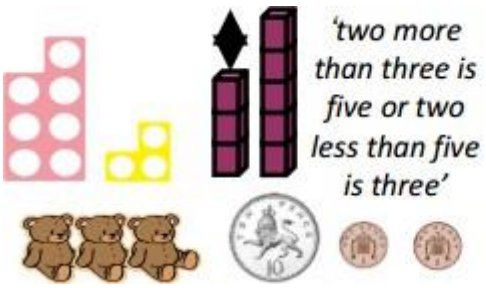
Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 3 then counting on 2.

Children develop an understanding of addition using Part-Part-Whole method, starting with practical hands-on learning with concrete materials progressing through to an abstract model.

Concrete	Pictorial	Abstract
 =  =   Use cubes to add two numbers together as a group or in a bar. 	  Use pictures to add two numbers together as a group or in a bar.  	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

Children learn to add by making 10 first (regrouping to 10) then adding the remainder.

9 + 3 = 12  6 + 5 = 10 	 3 + 9 = 9 + 5 = 14 	9 + 3 = 12 9 +1 = 10 10 + 2 = 12



Fluency:

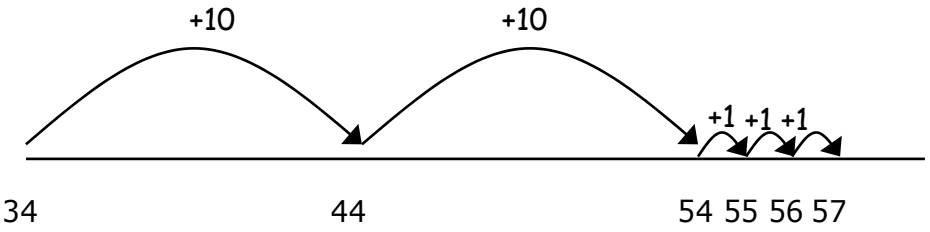
- Count forwards, to and across 100, beginning with 0 or 1 or from any given number Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 …
- Represent and use number bonds up to 20 (establish addition and subtraction as related operations)
- Find one more than a number
- Find ten more than a number
- Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

Year 2:

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

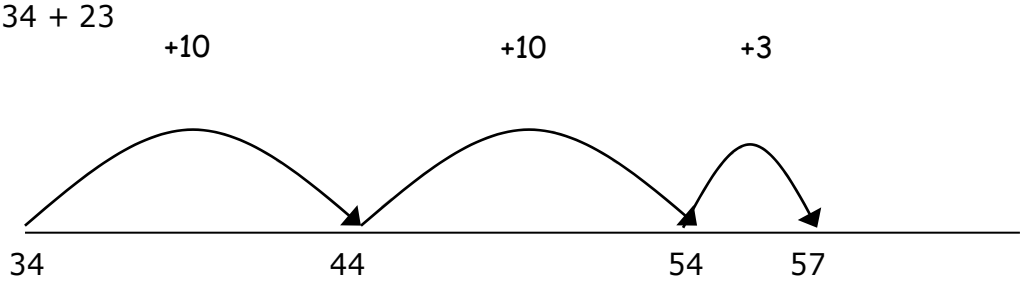
First counting on in tens and ones.

34 + 23 = 57



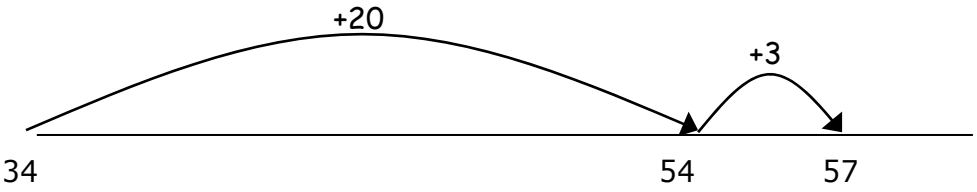
Then show children how to be more efficient adding the units in one jump (by using the known fact 4 + 3 = 7).

34 + 23

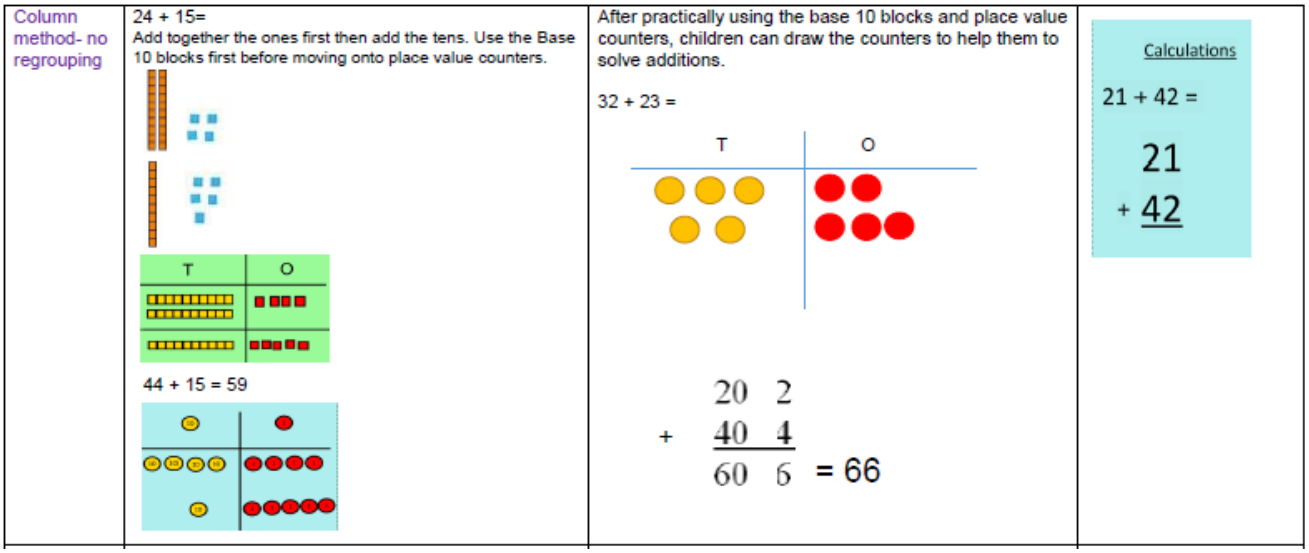


Followed by adding the tens in one jump and the units in one jump.

34 + 23 = 57

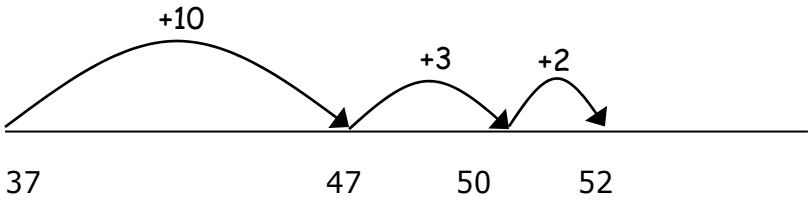


Children are taught addition using column method partitioning into place value.

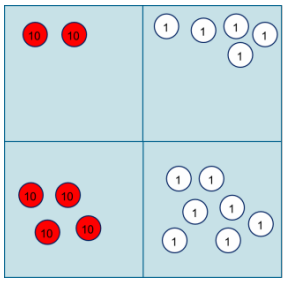


Encourage partitioning to bridge through a ten using a number line.

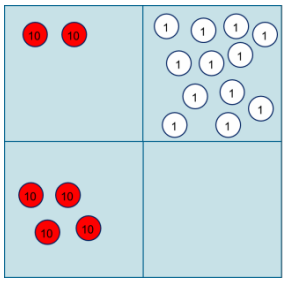
37 + 15 = 52



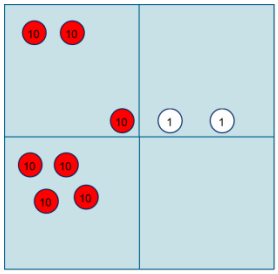
Children are introduced to regrouping (exchanging) using column method.



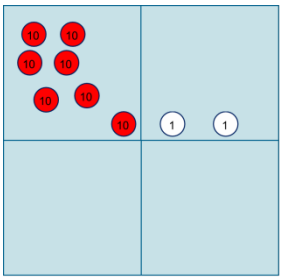
$$\begin{array}{r} 25 \\ +47 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ +47 \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ {\scriptstyle 1} \end{array}$$



$$\begin{array}{r} 25 \\ +47 \\ \hline 72 \\ {\scriptstyle 1} \end{array}$$

Fluency:

- Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20
- Use knowledge to derive and use number facts up to 100
- Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

Year 3:

Children will use informal expanded methods alongside compact column methods.

$$\begin{array}{r} 40 + 1 \\ +\ 20 + 8 \\ \hline 60 + 9 = 69 \end{array}$$

Expanded recording without exchange

$$\begin{array}{r} 40 + 3 \\ 20 + 8 \\ \hline 70 + 1 \quad = 71 \\ {\scriptstyle 10} \end{array}$$

Expanded recording with exchange

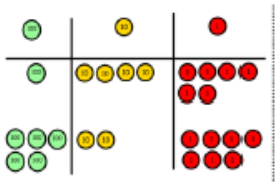
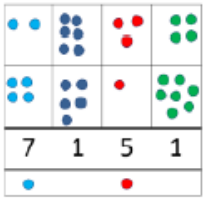
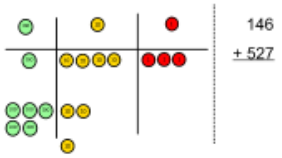
$$\begin{array}{r} 100 + 40 + 1 \\ +\ 100 + 20 + 8 \\ \hline 200 + 60 + 9 = 269 \end{array}$$

Expanded recording

$$\begin{array}{r} \text{HTU} \\ 141 \\ +\ 128 \\ \hline 269 \end{array}$$

Compact (column) recording

To support the transition a variety of concrete resources are used for teacher to model and children to select to use independently – money, counters, base 10, numicon, etc.

Column method- regrouping	Make both numbers on a place value grid.  146 + 527	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 2634 + 4517  7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 5 40 8 60 13 = 73 536 + 85 621 1 1
Column method- regrouping cont'd	Add up the units and exchange 10 ones for one 10. 146 + 527 = 673  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		As the children move on, introduce decimals with the same number of decimal places and different number of decimal places. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 . 3 6 1 . 0 8 0 + 5 . 7 7 0 1 . 3 0 0 9 . 5 1 1 2 2

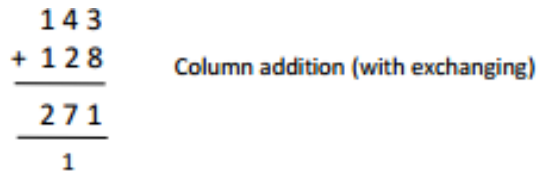
Fluency:

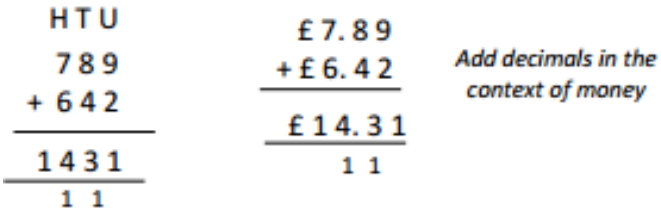
- Mentally add HTU + ones, HTU + tens, HTU + hundreds.
- Count in ones, tens and hundreds maintaining fluency through varied and frequent practice
- Count from 0 in multiples of 4, 8, 50 and 100
- Find 10 or 100 more than a number

- Perform mental calculations with two digit numbers, the answer could exceed 100

Year 4:

Children extend the regrouping (exchange) method to numbers with at least four digits and decimals.





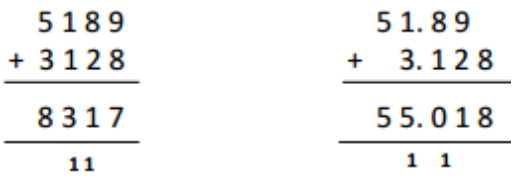
Compact (column) recording

Fluency:

- Count in 6s, 7s, 9s, 25s and 100s
- Find 1000 more than a number
- Perform mental calculations with increasingly large numbers to aid fluency

Year 5:

Children should extend the carrying method to numbers with more than four digits and decimals.



Column addition (with exchanging)

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency:

- Count forwards in powers of ten up to 100000
- Count forwards in positive and negative whole numbers through zero
- Practise mental calculations with increasingly large numbers
- Practise fluency of written methods

Year 6:

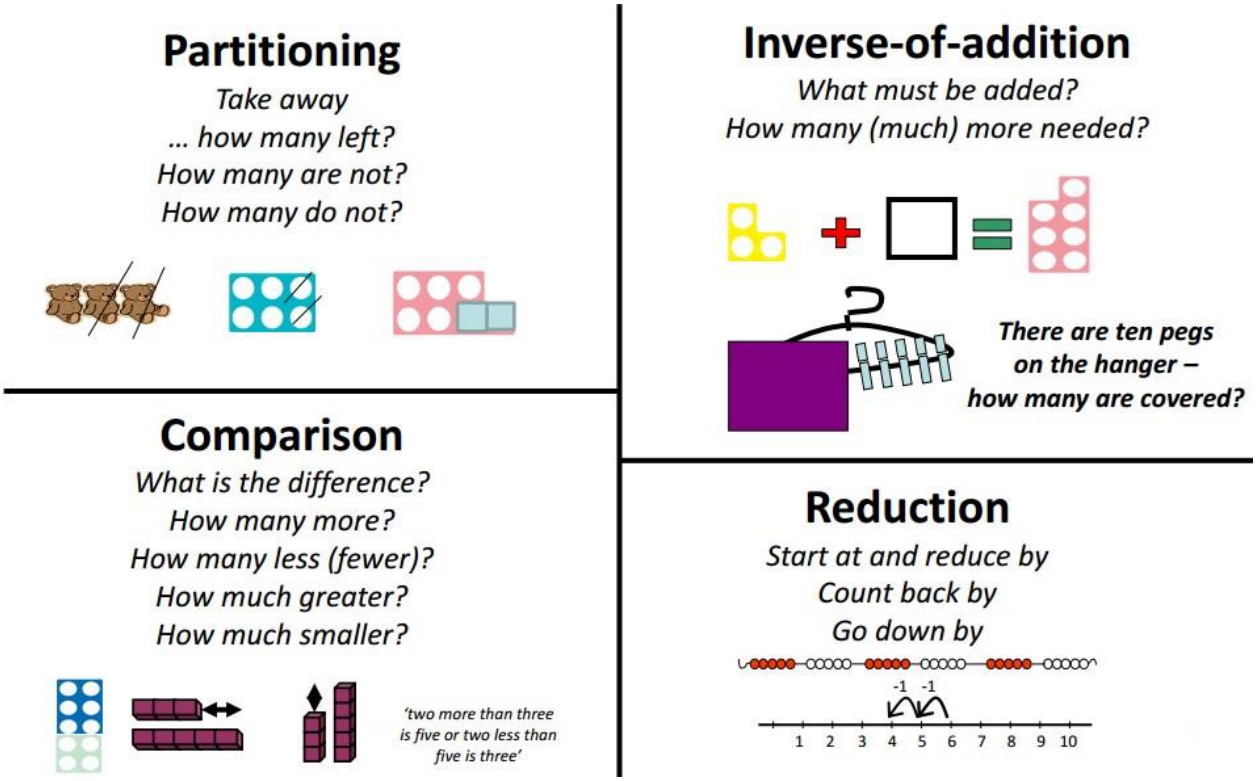
It is also essential that children acquire a range of effective mental calculation strategies, including using the relationship between addition and subtraction

Fluency:

- Count in tens and hundreds increasing fluency of order and place value
- Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency.

SUBTRACTION

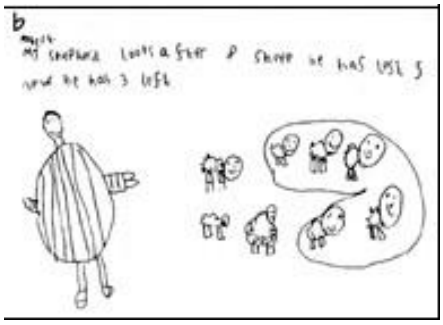
Subtraction is the inverse of addition. At its most basic, subtraction is taking one number away from another. Children experience a range of subtraction problems, using amounts and calculating differences between numbers.



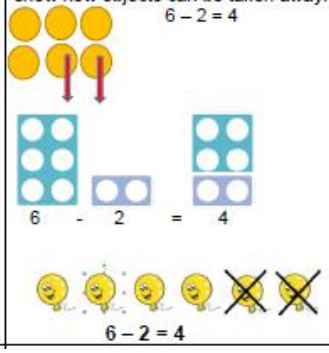
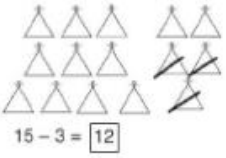
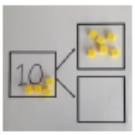
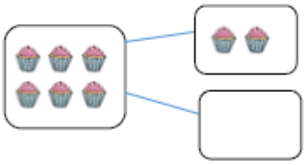
PROGRESSION THROUGH CALCULATION STRATEGIES

Year 1:

Children develop a mental picture of the number system to use for calculation. They develop ways of recording calculations using pictures etc.



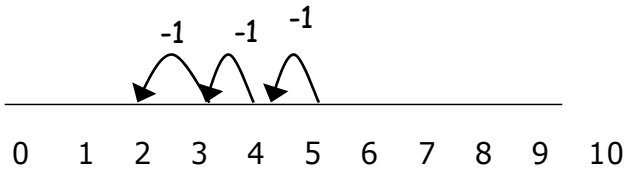
Children develop an understanding of subtraction, starting with practical hands-on learning with concrete materials progressing through to an abstract model, including the Part-Part-Whole model.

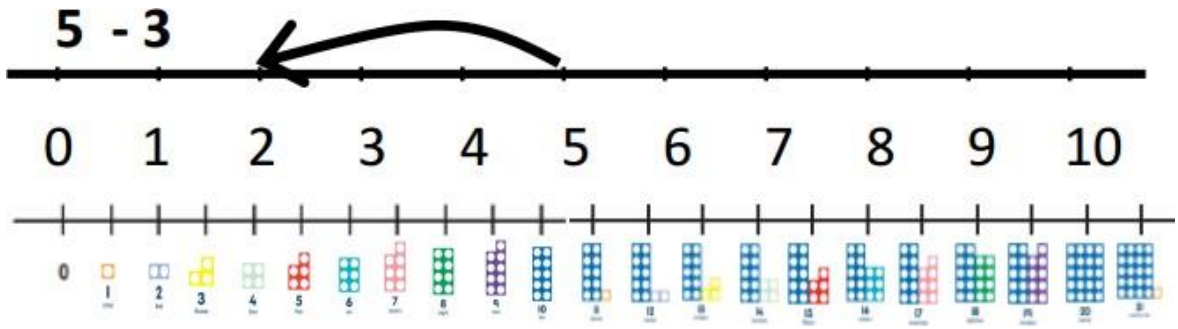
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  	6 – 2 = 4 15 -3= 12
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part part whole model.

They use number lines and practical resources to support calculation, including modelling of empty number lines.

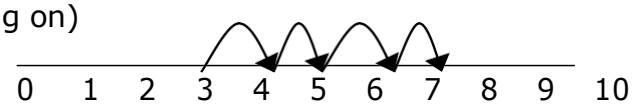
5 – 3 = 2

(counting back)

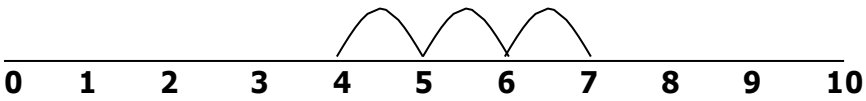


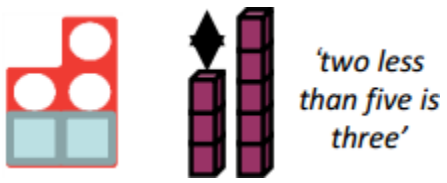


7 – 3 = 4
(counting on)



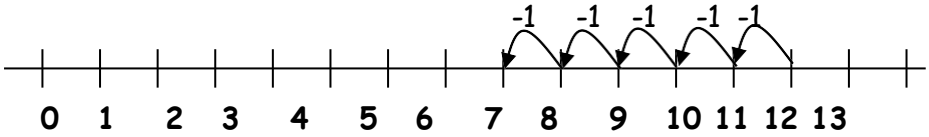
The number line should also be used to show that 7 - 3 means the 'difference between 7 and 3' or 'the difference between 3 and 7' and how many jumps they are apart.





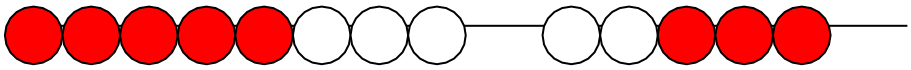
Children use number lines to support their own calculations - using a number line to count back/forward in ones.

12 – 5 = 7



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8



Fluency:

- Count backwards (including crossing 100) any given number
- Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10
- Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations)
- Find one less than a number
- Find ten less than a number Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern

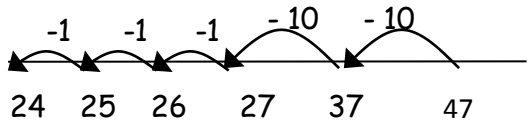
Year 2

Children will extend the use of number lines to include the using empty number lines to support calculations.

Counting back

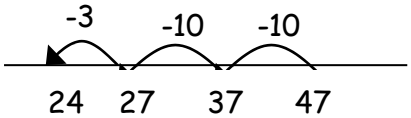
First counting back in tens and ones.

47 – 23 = 24



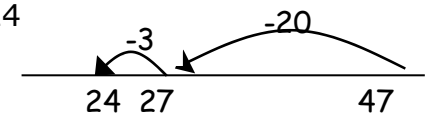
Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

47 – 23 = 24



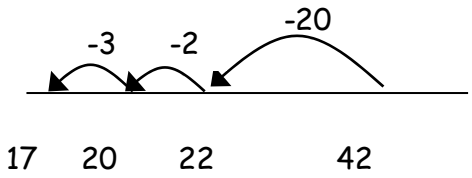
Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24



Bridging through ten can help children become more efficient. 42 –

25 = 17



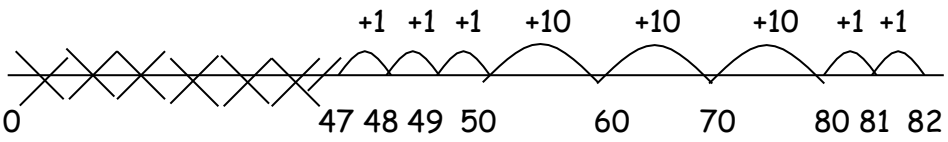
Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc., it can be more efficient to count on.

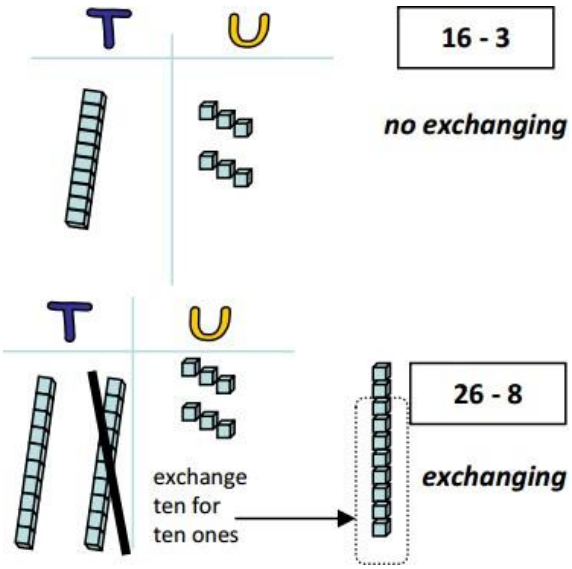
Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

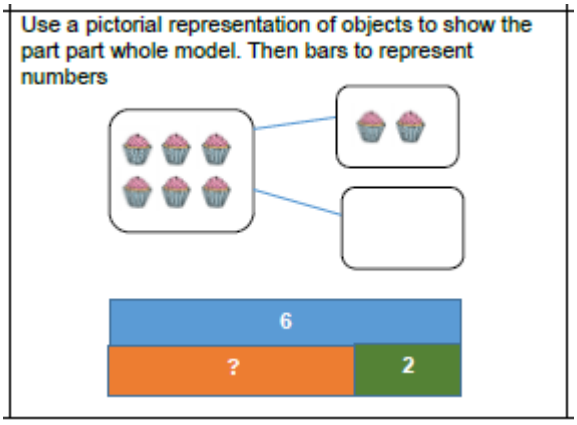
82 - 47



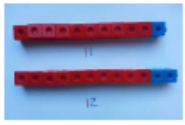
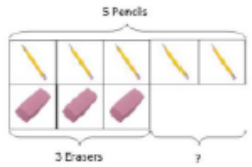
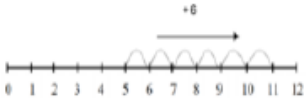
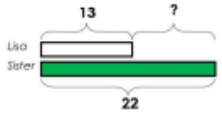
Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences



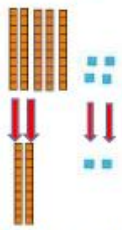
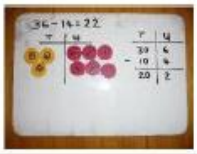
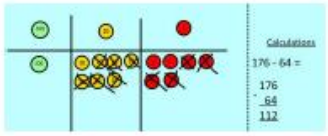
The Part-Part-Whole diagram is developed into an early understanding of bar method to represent the numbers.



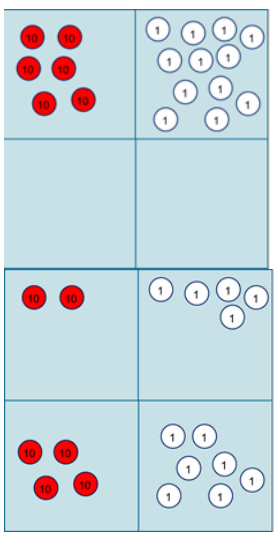
Children compare amounts and objects to find the difference.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference 	Count on to find the difference between 5 and 11.  Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

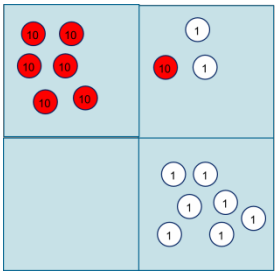
Similarly to the progression for addition, subtraction is taught using column methods, first without regrouping, and then with (exchanging).

Column method without regrouping	Use Base 10 to make the bigger number then take the smaller number away. 54 – 22 = 32  Show how you partition numbers to subtract. Again make the larger number first. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	47 – 24 = 23 407 -204 203 = 23 This will lead to a clear written column subtraction. 

A model involving the use of 'place value counters' can help in moving from expanded to compact methods. Beginning with base-10 equipment and/or place value arrow cards; and then using place value counters alongside expanded written methods to lead to formal written algorithms.



$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}2 \\ -\,4\,7 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}2 \\ -\,4\,7 \\ \hline 5 \end{array}$$

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}2 \\ -\,4\,7 \\ \hline 2\,5 \end{array}$$

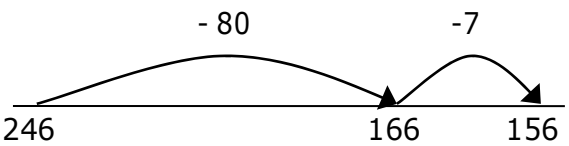
Fluency:

- Practise addition and subtraction facts to 20 Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20
- Use known facts to 20 to derive new facts e.g. 3 + 7 – 30 + 70
- Use knowledge to derive and use subtraction number facts up to 100

Year 3:

Children will continue to use empty number lines with increasingly large numbers; subtracting the number they are taking away in convenient and partitioned steps. In other cases, they step up from the smaller number to the large number.

246 – 87 = 246 – 80 – 7



Children will begin to use informal, expanded pencil and paper methods

The expanded method leads children to the more compact method so they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place-value

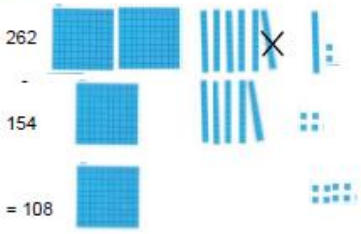
Expanded Method: Partitioning

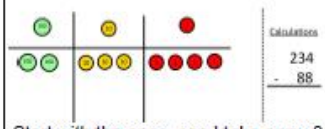
189 – 57

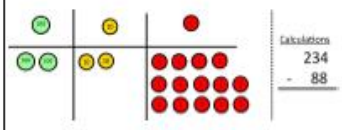
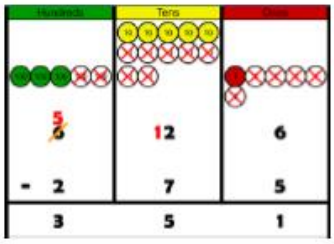
100 + 80 + 9
<u>50 + 7</u>
<u>100 + 30 + 2</u> = 132

This process should be demonstrated using place value arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

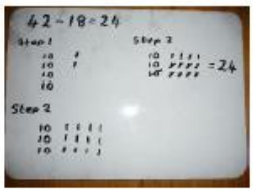
Models and images should always be used in developing conceptual understanding alongside arithmetic proficiency.

<table>
<tr>
<td>Column method with regrouping</td>
<td>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.
262 – 154 H T U
 
262
-
154
= 108

Make the larger number with the place value counters
 
Calculations
234
- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.
 
Calculations
234
- 88</td>
<td>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.
 
Hundreds Tens Ones
5 6 12 6
- 2 7 5
3 5 1

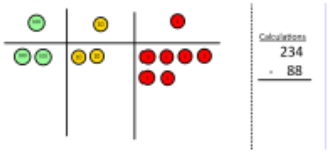
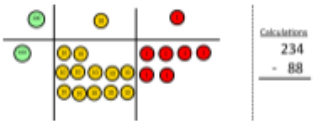
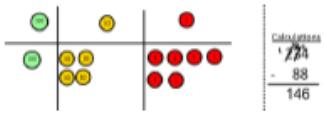
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.
 
42 - 18 = 24
Step 1
Step 2
Step 3
= 24</td>
<td> 
836-254=582
800 30 6
- 200 50 4
500 80 2

Children can start their formal written method by partitioning the number into clear place value columns.
 
728-582=146
700 20 8
500 80 2
200 60 6

Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.
5 12 1
2 6 3 . 0
- 2 6 . 5
2 3 6 . 5</td>
</tr>
</table>

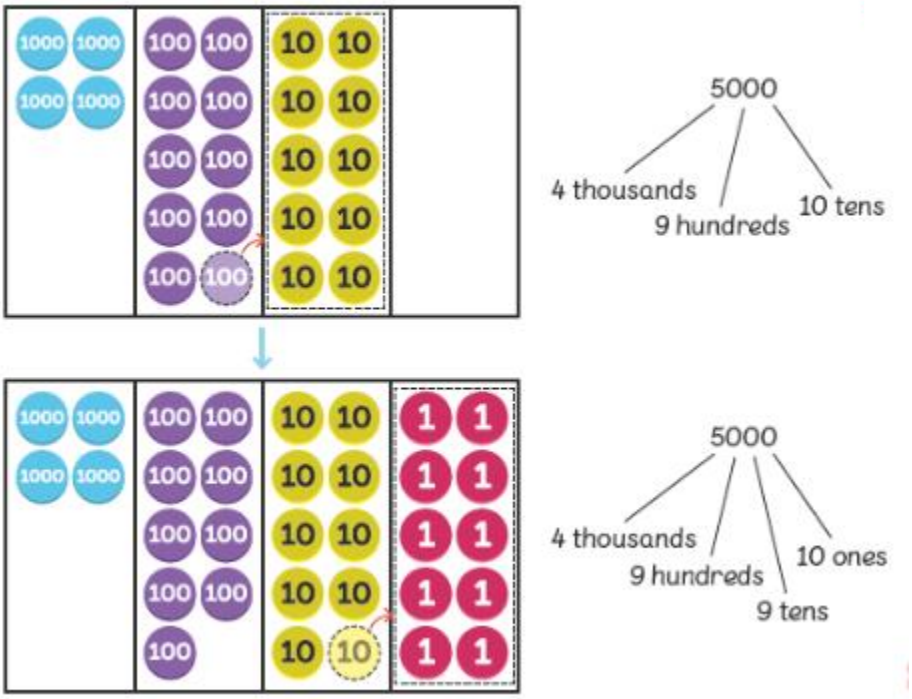
Column method with regrouping cont'd	Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Fluency:

- Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice
- Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267
- Mentally subtract HTU + ones, HTU + tens, HTU + hundreds Perform mental calculations with two-digit numbers
- Find ten and a hundred less than a number with up to three-digits

Year 4:

Children will have increased proficiency with formal written methods, and these are encouraged as the preferred method. Expanded methods will still be used with the introduction of numbers with higher place values and when partitioning numbers in a range of ways allowing for visual exchange.



Fluency:

- Count back in 6, 7, 9, 25 and 1000
- Count back through zero to include negative numbers
- Find 1000 less than a number
- Continue to practise mental calculations with increasingly large numbers to aid fluency.

Year 5/6

Children will still use a full range of concrete-pictorial-abstract resources and imagery, as introduced further down the school, however they will be able to independently choose the strategy that best suits their own learning approach.

Children will have a range of mental and written methods, including formal columnar methods, which they can apply efficiently. They will have opportunities to apply these through varied and frequent practice with increasingly complex problems.

Ensure children can solve calculations where zero is a place holder

```
  1.4 8
- 1.2 1
-------
  0. 2 7
-------
```

Column subtraction (no exchanging)

Column subtraction (with exchanging)

```
  6 11 1
  7.2 3
- 3.6 7
-------
  3.5 6
-------
```

Fluency:

- Count backwards in powers of ten up to one million
- Count backwards in positive and negative whole numbers through zero
- Practise mental calculations with increasingly large numbers
- Undertake mental calculations with increasingly large numbers and more complex calculations

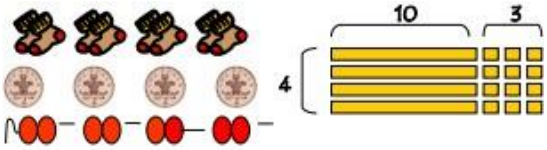
MULTIPLICATION

Multiplication is **defined** as to calculate the result of repeated additions of two numbers. An example of **multiplication** is 4 times 2 equals 8.

It is essential that all children learn their times tables facts. They are used in many areas of maths and need to be learnt by heart. Children need to understand that multiplication is based on repeated addition. Once this is understood, various methods of helping them to multiply 2 numbers and derive an accurate answer are taught.

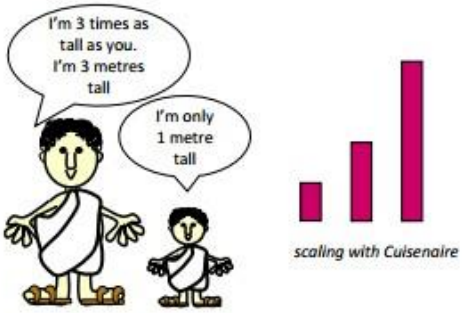
Repeated addition

So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)



Commutative law

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal

4 x 2 is the same as/equal to 2 x 4

MULTIPLICATION AND DIVISION EXPECTATIONS BY YEAR GROUP

Year 1:

- Related statutory requirements
- solve one-step problems involving multiplication and division, by calculating the answer
- using concrete objects, pictorial representations and arrays with the support of the teacher
- count in multiples of twos, fives and tens

Year 2:

- Related statutory requirements

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in
- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward

Year 3:

Related statutory requirements:

- *recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables*
- *write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and* ***progressing to formal written methods***
- *solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.*
- *count from 0 in multiples of 4, 8, 50 and 100;*

Year 4:

Related statutory requirements:

- count in multiples of 6, 7, 9, 25 and 1000

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- **multiply two-digit and three-digit numbers by a one-digit number using formal written layout**
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
- children continue to practise recalling and using multiplication tables and related division facts to aid fluency

Year 5:

Related statutory requirements:

- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- **multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**
- multiply and divide numbers mentally drawing upon known facts
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Year 6:

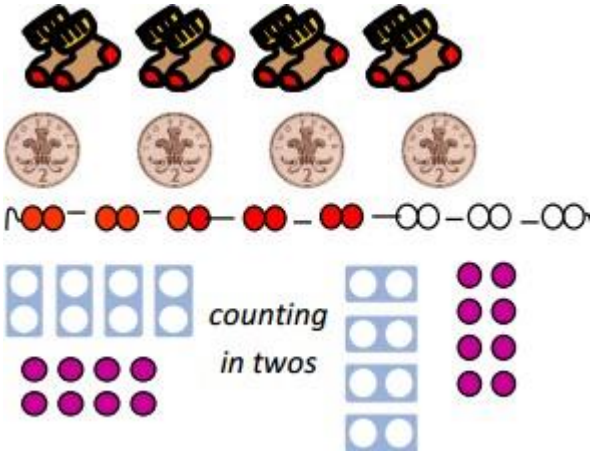
Related statutory requirements:

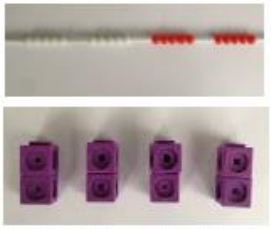
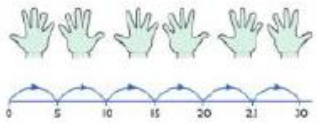
- **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication**

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

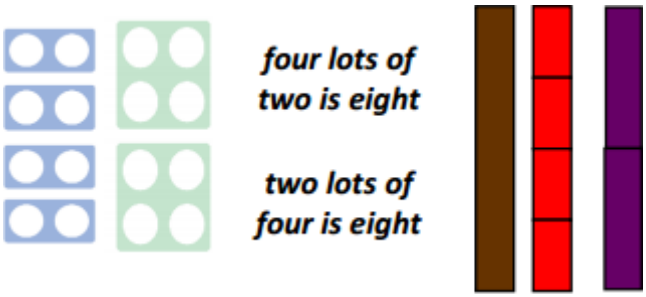
Year 1:

Children will experience equal groups of objects and will count in 2s 5s and 10s. They will work on practical problem solving activities involving equal sets or groups.

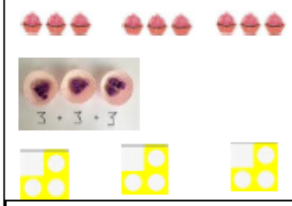
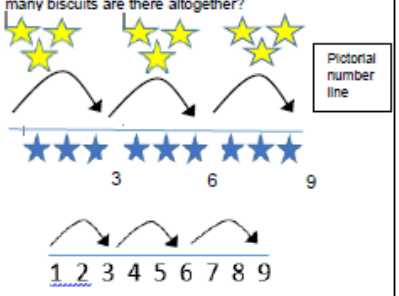


Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

Children make connections (with adult support) between arrays, number patterns, and counting in twos, fives and tens.



Multiplication is linked to repeated addition.

Repeated addition	 Use different objects to add equal groups.	There are 3 plates. Each plate has 3 biscuits on it. How many biscuits are there altogether?  Pictorial number line	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

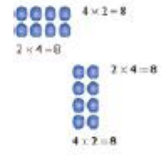
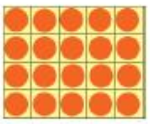
Fluency:

- Count in twos, fives and tens from different multiples e.g. 6, 8, 10, 12 etc
- Emphasise number patterns
- Double number and quantities

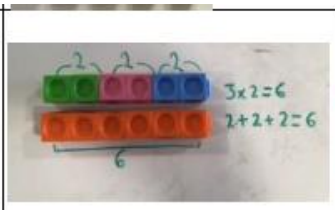
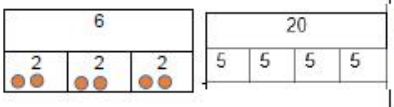
Year 2:

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

Arrays are a very powerful and versatile image to develop understanding in both multiplication and division.

Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find commutative multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

The use of bar models for representing multiplication is introduced, especially when solving word problems.

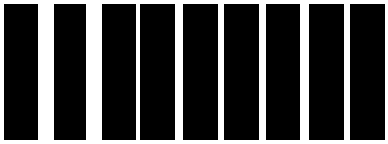
Bar models representing multiplication.		 2 + 2 + 2 = 6 3 x 2 = 6 5 + 5 + 5 + 5 = 20 4 x 5 = 20	3 x 2 = 6 4 x 5 = 20

Fluency:

- Count in twos, threes, fives from zero and tens from any number e.g. 6, 8, 10, 12 etc
- Emphasise number patterns Introduction to multiplication tables.
- Practise to become fluent in multiplication facts for 2, 3, 5 and 10
- Solve multiplication problems mentally

Year 3:

Children can model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

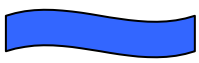


9 x 4 = 36

4 x 9 = 36

Children will also develop an understanding of scaling as an important element of multiplicative relationships

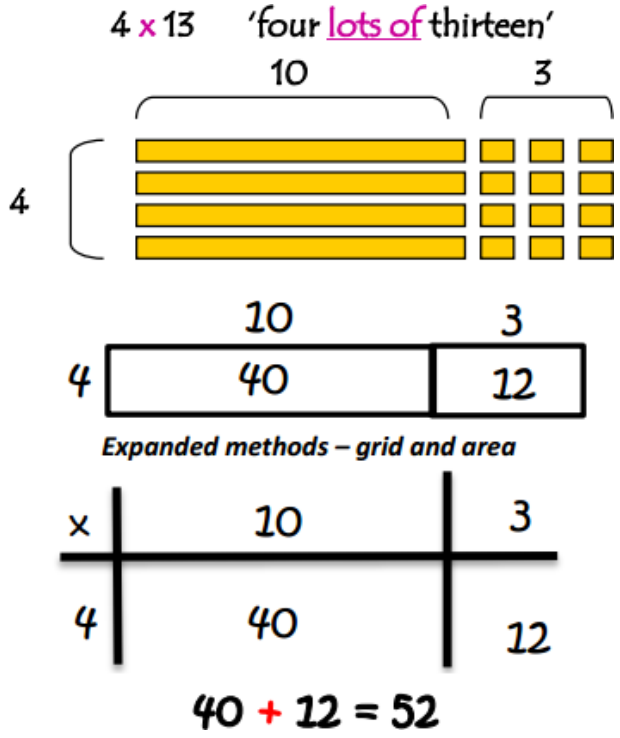
e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm

20 cm

Children will continue to use arrays where appropriate, leading into the grid method of multiplication as a step towards formal written method for short multiplication.

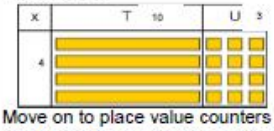


Grid Method

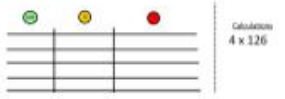
Show the link with arrays to first introduce the grid method.

4 rows of 10
4 rows of 3

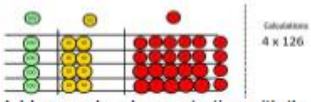
Move on to using Base 10 to move towards a more compact method.
4 rows of 13



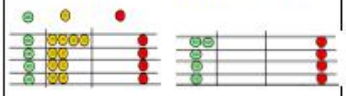
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



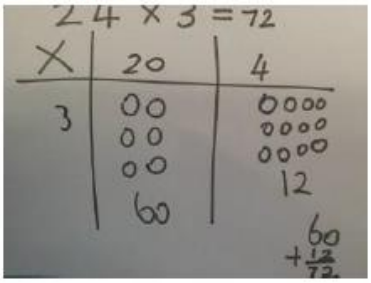
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer = 504

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying 2, 3 and 4 digit numbers by one digit number and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

100
80
30
+ 24
234

18 x 13 = 234

Then progress to 2 digit by 3/4 digit number.

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

1342 x 18 = 24156

10000
8000
3000
2400
400
320
20
+ 16
24156

Move to decimals with grid.
e.g 4.9 x 3

x	4	0.9
3	12	2.7

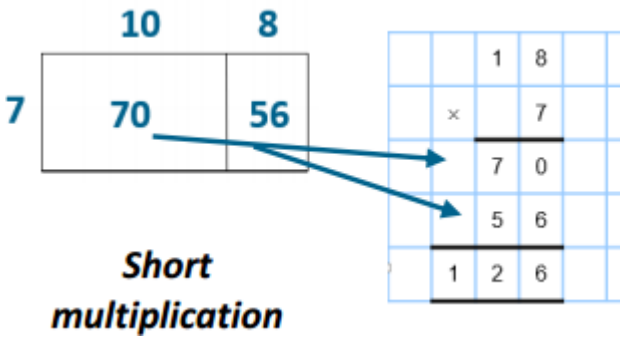
12
+ 2.7
14.7

5 x 6.23

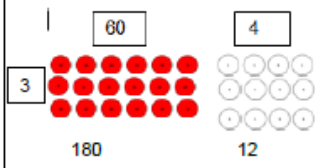
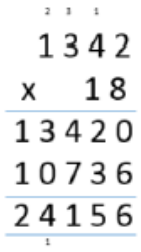
	6	0.2	0.03
5	30	1.0	0.15

30.00
+ 1.00
0.15
31.15

Children will move toward using formal written methods.



When short multiplication is introduced it should be modelled alongside the grid method so that children understand how one is a different representation of the other.

Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  64 x 3 = 180 + 12 = 192 It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768  This moves to the more compact method. 

Fluency:

- Count from 0 in multiples of 4, 8, 50 and 100
- Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100
- Practise mental recall of multiplication tables – 3, 4 and 8x times tables
  Connect the 2, 4 and 8 times tables using doubling
- Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. 4 x 4 x 12 = 12 x 4 x 5 = 12 x 20

Year 4:

Children will continue to develop an understanding of formal written method for multiplication including multiplication of three numbers and factors.

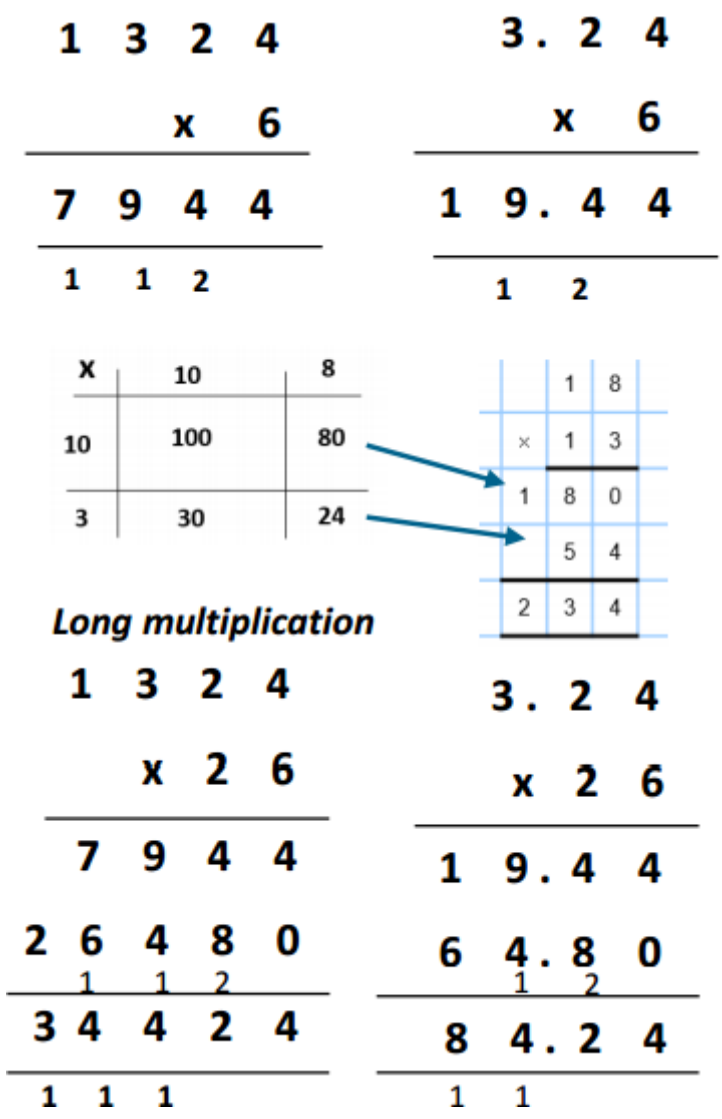
In Year 4 children will take the Multiplication Test for all multiplication facts up to 12X12.

Fluency:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall and use multiplication facts up to 12 x 12 with increasing fluency
- Derive multiplication facts with up to three-digits
- Recognise and use factor pairs and commutativity in mental calculations
- Use the distributive law
- Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. 2 x 6 x 5 = 10 x 6

Year 5:

Moving on to use longer multiplication sums (with decimal places) to represent a range of measures and amounts.



Fluency:

- Count forwards in steps of powers of 10 from any given number up to 1 000 000
- Practise and extend use of formal written method of short multiplication
- Apply all multiplication tables frequently.
- Commit them to memory and use them confidently to make larger calculations
- Multiply numbers mentally drawing upon known facts

Year 6:

Children will need to confidently and efficiently use formal written methods.

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 24 \\ \times\ 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

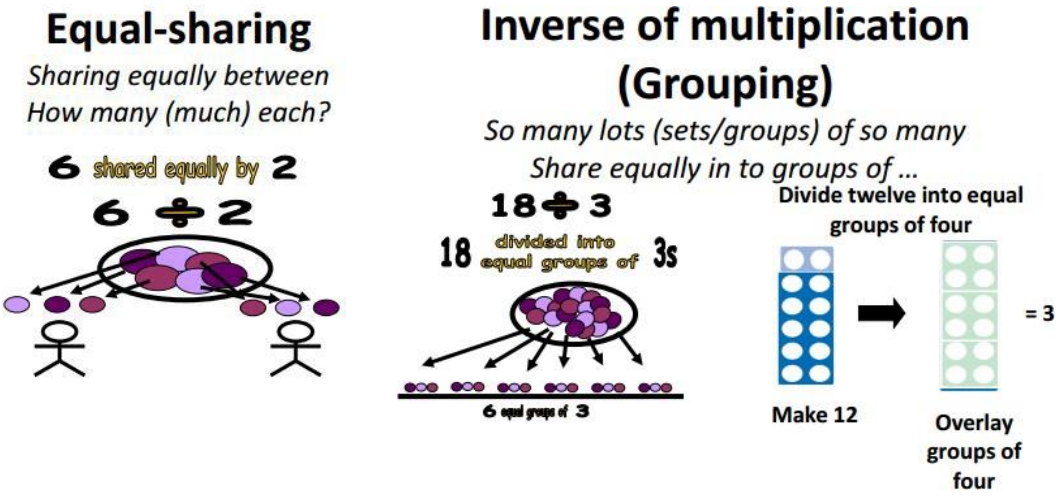
Fluency:

- Undertake mental calculations with increasingly large numbers
- Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency.

DIVISION

Children will experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement.

Division is the action of separating something into parts or the process of being separated. Also it can be interpreted as the mathematical process of finding out how many times one number is contained in another. It is the inverse of multiplication and fluency with multiplication tables is invaluable when approaching division problems. Other words that children may have experienced when referring to division are sharing, grouping, dividing, lots of and numbers of.



Ratio structure

comparison
inverse of scaling structure of multiplication
scale factor (decrease)

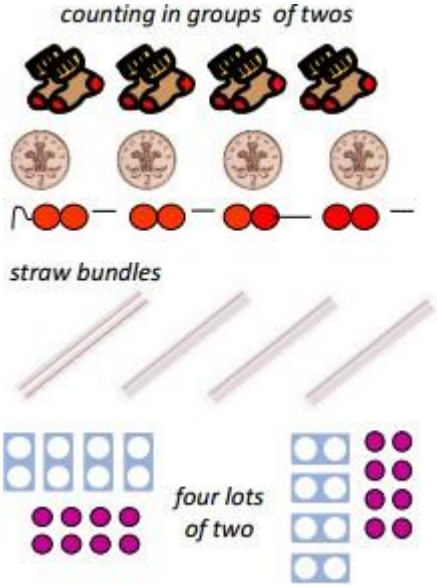
Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

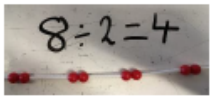
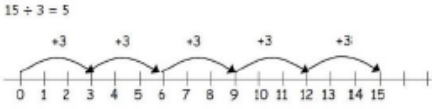
PROGRESSION THROUGH CALCULATIONS FOR DIVISION

Year 1:

Children will understand equal groups and share items out in play and problem solving. Children will also link counting in groups to groups of numbers.



Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?  10 ÷ 2 = 5 	Children use pictures or shapes to share quantities. They can draw the number of groups they are spliiting into first. 8 ÷ 2 = 4 	Share 9 buns between three people. 9 ÷ 3 = 3

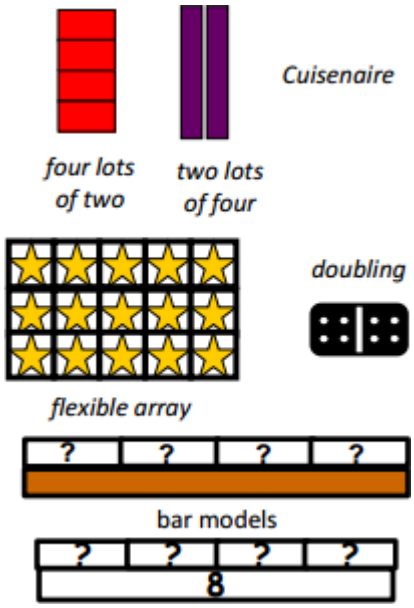
Division as grouping	 10 ÷ 5 = 2 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 15 ÷ 3 = 5 	

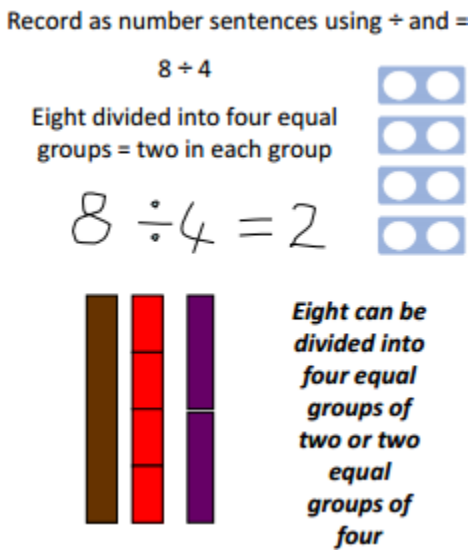
Fluency:

- Count in twos, fives and tens from different multiples e.g. 6, 8, 10, 12 etc
- Emphasise patterns
- Find simple fractions eg half and quarter, of objects, numbers and quantities

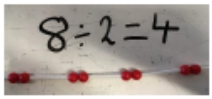
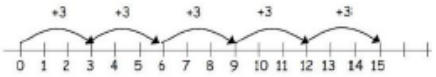
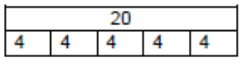
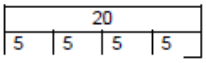
Year 2:

Children will develop their understanding of division and use jottings to support calculation. Children will also begin to record division problems as number sentences using ÷ and =.

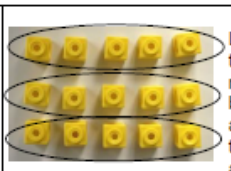
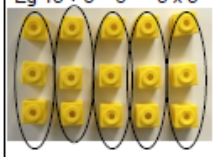
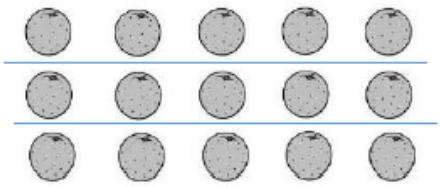




Division as grouping is developed to include the use of bar model images to solve word problems.

Division as grouping	 10 ÷ 5 = 2 Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 15 ÷ 3 = 5  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  How many 5s in 20? 20 ÷ 5 = ? 5 x ? = 20  How many 4s in 20? 20 ÷ 4 = ? 4 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Children will make the inverse link between multiplication and division through creating arrays.

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

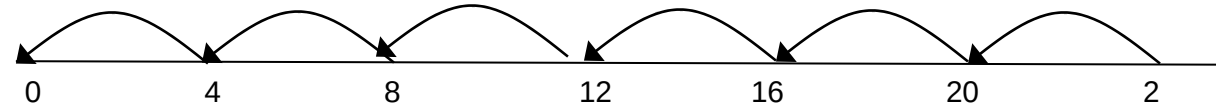
Fluency:

- Count back in twos, threes, fives from zero and tens from any number e.g. 12, 10, 8, 6 etc
- Emphasise patterns
- Connect ten times table to place value and five times table to divisions on a clock face Introduction to multiplication tables.
- Practise to become fluent in division facts for 2, 5 and 10
- Solve division problems involving grouping and sharing

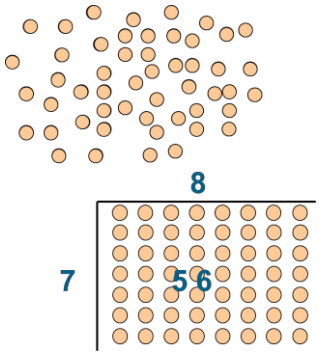
Year 3:

Children will continue to make the link between repeated additions on a number line and multiplication.

24 ÷ 4 = 6



Children will have a range experiences in partitioning numbers into multiples of the divisor before progressing to short division.



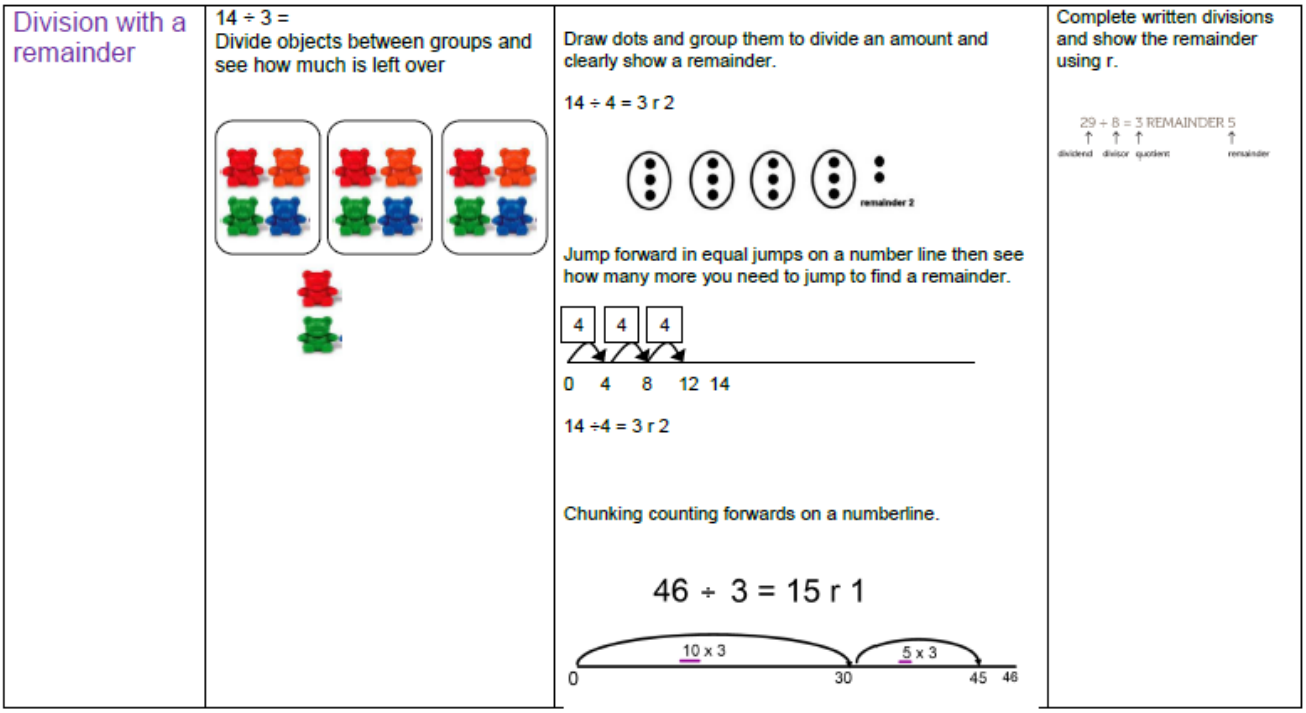
Either:

- How many 7s can I see? (grouping)

Or:

- If I put these into 7 groups how many in each group? (sharing)

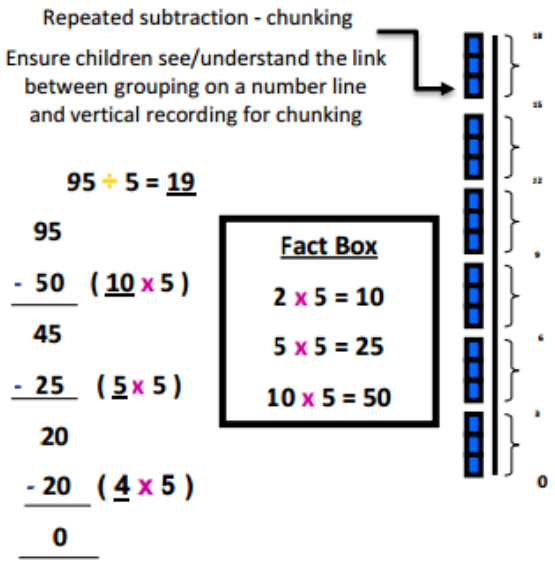
Children will explore what happens when an amount cannot be divided equally into groups and how this can be recorded.



The use of arrays will continue to be useful in developing understanding of division. Children will, however, need to be transitioned toward the use of the standard algorithm for short division.

The use of chunking will enable children to make the link between arrays and the

short multiplication method.



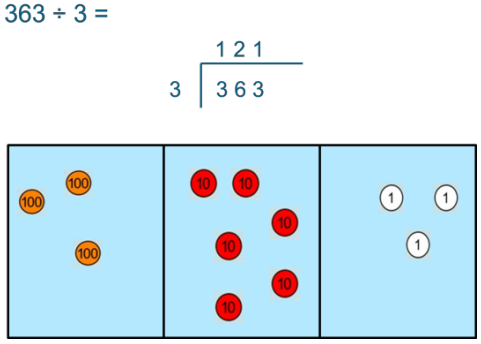
The use of place value counters can be very helpful in understanding how the standard algorithm works.

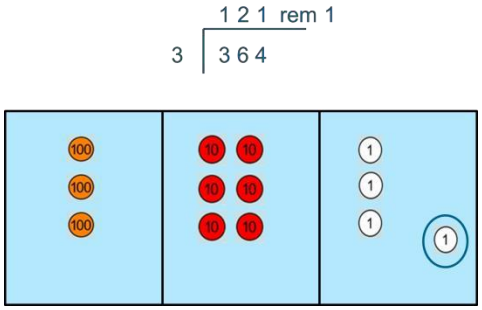
Fluency:

- Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)
- Write and calculate mathematical statements for division using what is known
- Use division facts to derive related division facts e.g. using 6 ÷ 3 = 2 to work out 60 ÷ 3 = 20

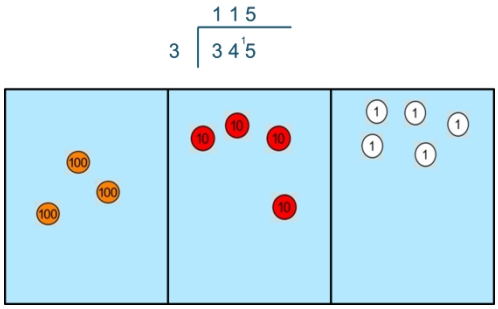
Year 4:

The use of chunking is continued to be used to develop an understanding of short multiplication method, using place value counters etc for CPA representation.

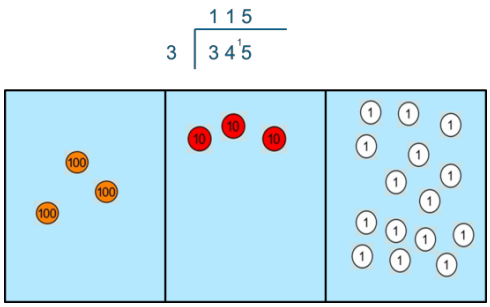




345 ÷ 3 =



345 ÷ 3 =



Fluency:

- Continue to practise recalling division facts for multiplication tables up to 12 x 12
- Practise mental methods and extend this to three-digit numbers for example 200 x 3 = 600 into 600 ÷ 3 = 200
- Use place value, known and derived facts to divide mentally, including dividing by 1
- Recognise and use factor pairs and commutativity in mental calculations

Year 5:

Introduce short division standard written method for division

98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\,9\ 8} \\ \quad 2 \end{array}$$

Answer: 14

432 ÷ 5 becomes

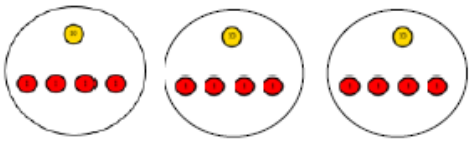
$$\begin{array}{r} 8\ 6\ r2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad 3 \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 4\ 5\ r1 \\ 11\overline{)\,4\ 9\ 6} \\ \quad 5 \end{array}$$

Answer: $45\frac{1}{11}$

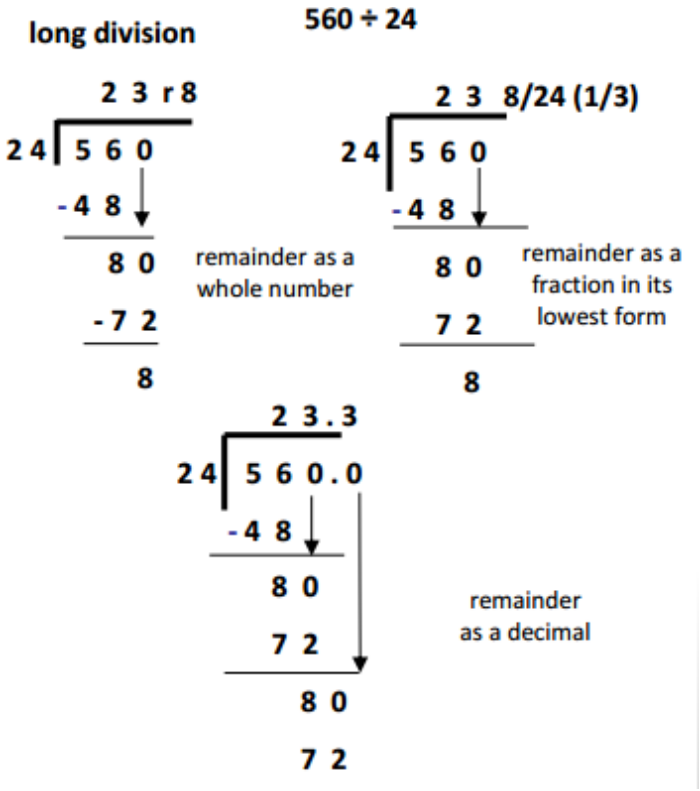
Short division		Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. 42 ÷ 3 = 14  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 2\ 1\ 8 \\ 4\overline{)\,8\ 7\ 2} \\ \quad 3 \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 8\ 6\ r\ 2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad 3 \end{array}$ Finally move into decimal places to divide the total accurately. $\begin{array}{r} 1\ 4\ .\ 6 \\ 35\overline{)\,5\ 1\ 1\ .\ 0} \\ \quad 16\quad 21 \end{array}$ What is £36.26 split between 23 people? $\begin{array}{r} 1.53 \\ 23\overline{)\,35.^{12}1^{19}9} \end{array}$ = £1.53

Fluency:

- Count backwards in steps of powers of 10 for any given number up to 1 000 000
- Count backwards with positive/negative whole numbers through zero
- Practise mental calculation with increasingly large numbers
- Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations

Year 6:

Extend long division



Fluency:

- Practise division for larger numbers, using the formal written methods of short and long division
- Continue to use all multiplication tables and division facts to maintain fluency
- Perform mental calculations, including with mixed operations and larger numbers